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The Deconfining Phase Transition in Lattice Quantum Chromodynamics

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We present a large-scale Monte Carlo calculation of the deconfining phase transition temperature in lattice quantum chromodynamics without fermions. Using the Wilson action, the transition temperature as a function of the lattice coupling g is consistent with scaling behavior dictated by the perturbative β function for $6/g^2 > 6.15$.

KEY WORDS: Quark-gluon phase transition; lattice quantum chromdynamics; Monte Carlo simulation.

INTRODUCTION

This presentation is based on some recent work of our large-scale collaboration to study lattice quantum chromodynamics at finite temperatures.[(1)]

In lattice QCD calculations one of the outstanding problems is to remove the lattice cutoff effects from physical quantities. This can only be accomplished with confidence in the scaling regime of the theory where the renormalization group β function is universal and known in perturbation theory.

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We believe that the deconfining phase transition with its first-order character may be the best tool to study the continuum limit of lattice QCD. The determination of the transition temperature is a unique test of the onset of scaling behavior, since the transition temperature T_c is free of cutoff dependent ultraviolet divergences. Locating T_c is relatively easy because the system undergoes a sharp first-order phase transition where rounding effects are small and controlled by finite-size scaling theory. On the basis of finite-size scaling theory for bulk properties we expect that thermodynamic quantities such as T_c can be studied on a smaller lattice than can correlations at the same value of the lattice coupling g.

At present these calculations can be carried out only when the effects of quark vacuum polarization are neglected. The hope is that the experience gained from these studies of the pure gauge theory will be useful when improved techniques and larger computing power make inclusion of dynamical fermions practical.

THE DECONFINING PHASE TRANSITION AND THE β FUNCTION

The transition temperature T_c of the deconfining phase transition is a measurable physical quantity in the continuum. In the scaling limit of lattice QCD, T_c is renormalization group-invariant; that is, $(d/da)\, T_c[g(a)] = 0$, where a is the lattice cutoff. Using the known two-loop perturbative β function

$$-\beta(g) = b_0 g^3 + b_1 g^5 + \cdots \tag{1}$$

with

$$b_0 = \frac{11}{16\pi^2}$$

and

$$b_1 = \frac{102}{(16\pi^2)^2}$$

T_c depends on the lattice coupling g as

$$T_c = \text{const}\, \frac{1}{a} \exp\left[-\int_0^g \frac{dg'}{\beta(g')} \right] \tag{2}$$

The constant in (2) must be determined from nonperturbative calculations of T_c in the scaling regime.

We use a Monte Carlo measurement of the function $aT(g)$ to determine the β function. The results are to be compared with the two-loop form of (1) to verify asymptotic scaling.

GLOBAL $Z(3)$ SYMMETRY AND THE DECONFINING TRANSITION

Gluon thermodynamics for finite temperatures is realized by lattices with spatial volume n_s^3 and temporal size n_t.[2,3] The temporal size n_t is identified with the inverse of the dimensionless function $\tau(g) = aT$. The spatial size n_s should be taken to infinity for fixed n_t in the thermodynamic limit. This is only approximately realized in Monte Carlo calculations, and finite-size effects become an important issue in the discussion.

The order parameter of the deconfining phase transition is the Polyakov loop $P(\mathbf{x})$ which is defined as

$$P(\mathbf{x}) = \mathrm{tr} \prod_{t=1}^{n_t} U_t(\mathbf{x}, t) \tag{3}$$

where U_t is an $SU(3)$ matrix along a timelike link at spatial location $\mathbf{x}$ at time t.[4,5] We shall denote the spatial average of $P(\mathbf{x})$ by P and use it as the order parameter of the transition.

Lattice QCD at finite temperature has a global $Z(3)$ symmetry in addition to the local gauge symmetry associated with the color $SU(3)$ group. The action under the $Z(3)$ symmetry is invariant under multiplication of all timelike links on a single time slice by the same element of $Z(3)$. Under this symmetry transformation the order parameter transforms as

$$P \to zP \tag{4}$$

where z is a group element from $Z(3)$. In the low-temperature confined phase $P = 0$ and the $Z(3)$ symmetry is unbroken. In the high-temperature deconfined phase $P \neq 0$ and the symmetry is broken with a three-fold degeneracy.

The dynamics of the Polyakov loop is determined by a three-dimensional effective theory: by local gauge invariance all timelike links can be set to the unit $SU(3)$ matrix except on one time slice. By integrating out the spatial link variables, one can derive an effective $SU(3)$ spin model in three dimensions to describe the interaction of the time-like link variables in the time-slice.[6,7,8] If this effective $SU(3)$ spin model has short-range interactions, it is in the same universality class as the three-state Potts model in three dimensions and therefore a first-order transition is expected.

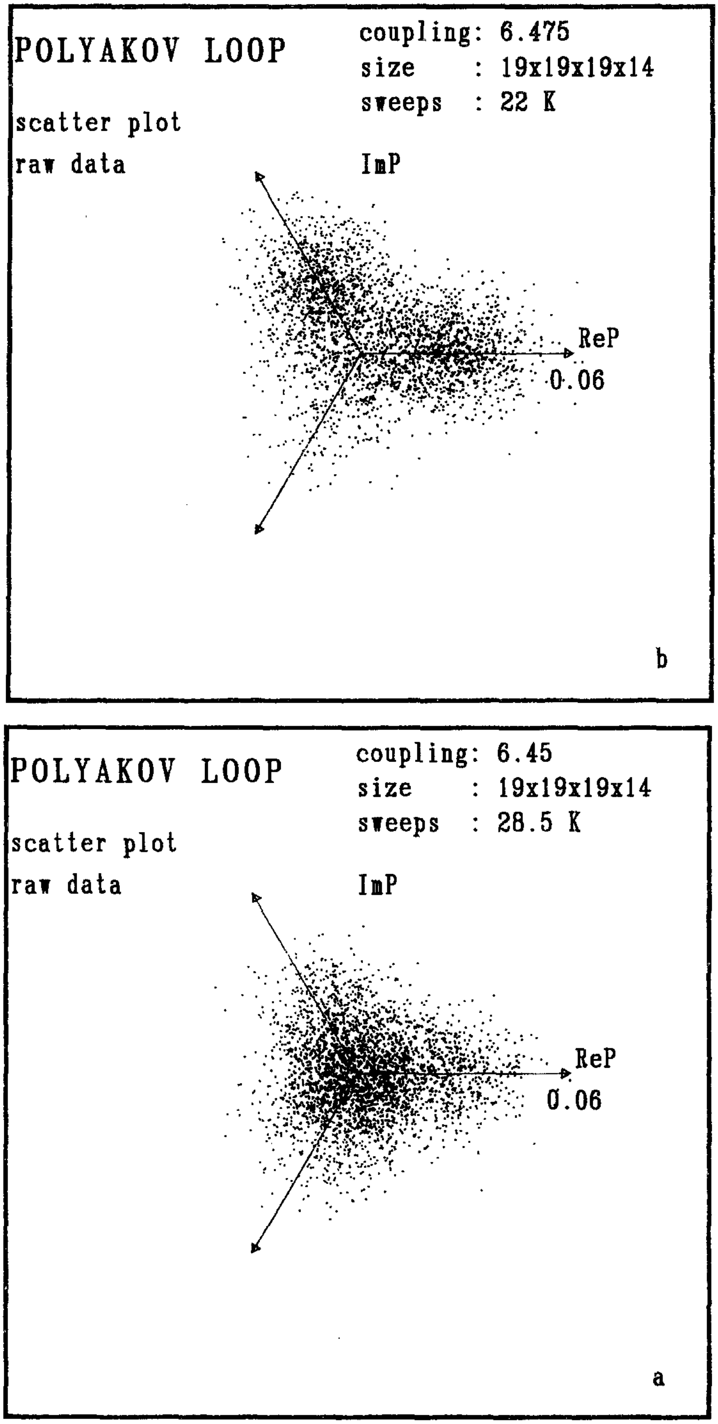

Fig. 1. Scatter plots of the Polyakov loop on $19^3 \times 14$ lattices at $6/g^2 = 6.45$, 6.475, and 6.5 showing the transition from the confined phase to the deconfined phase. The runs contain 28500, 22000, and 23750 sweeps, respectively, exclusive of warmups. The average Polyakov loop over the lattice is plotted at every tenth sweep.

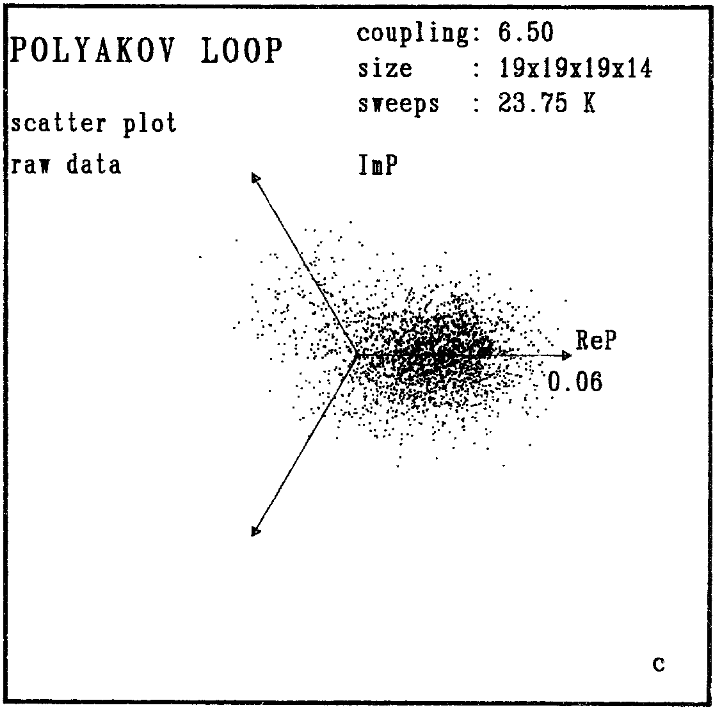

Fig. 1 (*continued*)

The high-temperature broken symmetry phase is identified as a deconfined phase because the free energy F_Q of an isolated external quark source, as related to P by

$$P \approx e^{-n_t a F_Q} \tag{5}$$

is finite when $P \neq 0$.

THE MONTE CARLO SIMULATION

In a recent paper,[9] a Monte Carlo calculation of the critical coupling g was reported for τ^{-1} ranging from 2 to 10. A pronounced nonscaling behavior was found in the coupling constant range $5.1 < 6/g^2 < 6.1$. A new very large-scale simulation is reported here in for τ^{-1} from 8 to 14.

We used the "Wilson" action, or the trace of the product of the link matrices around the elementary plaquettes, in the fundamental representation. The calculations were done one Cyber 205 supercomputers and on ST100 array processors. Different programs were used, one using the "quasi-heat-bath" method,[10] and the other the Metropolis method. The Cyber 205 code was running in 32-bit precision with 19 μs/link update

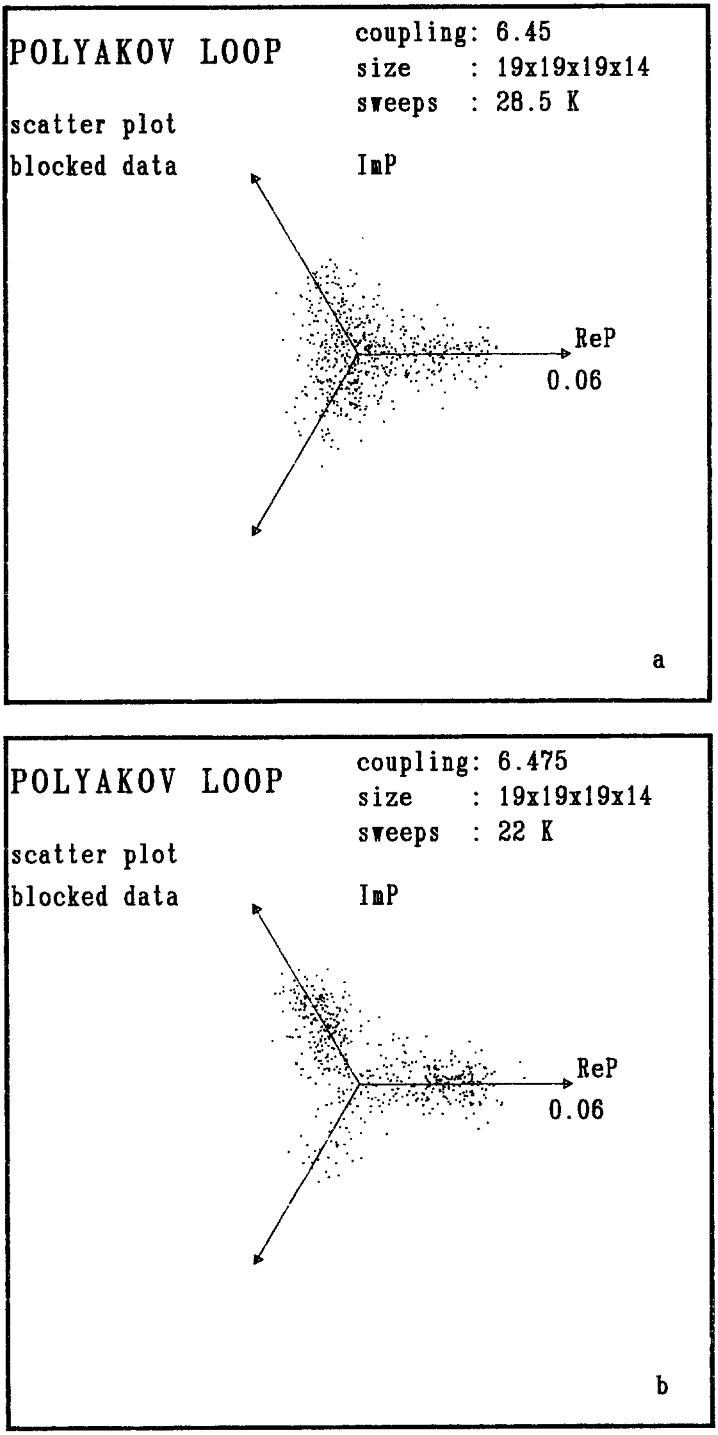

Fig. 2. The same data as in Fig. 1. Here five successive measurements have been averaged to smooth out the high-frequency fluctuations.

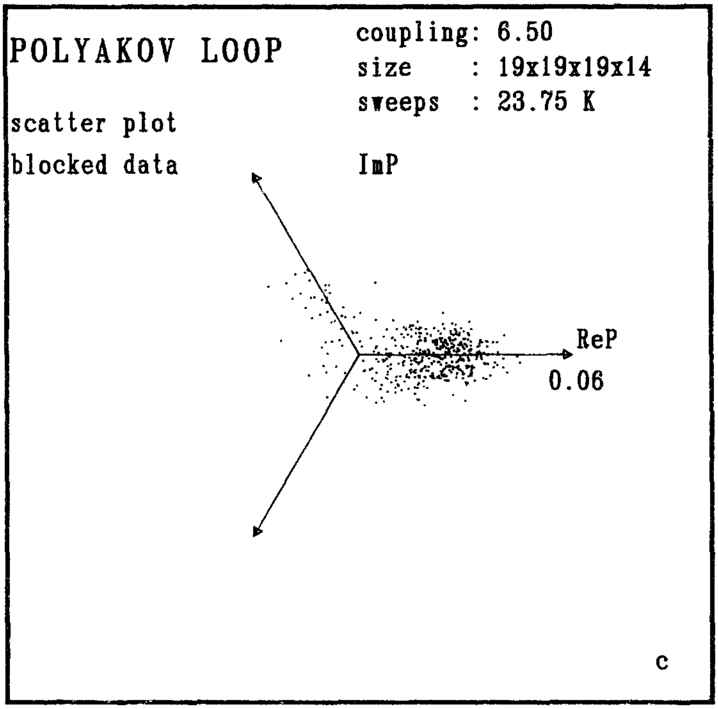

Fig. 2 (*continued*)

time using the quasi-bath algorithm. The ST100 code used 16-bit precision with 105 μs/link update time with 15 Metropolis hits/link including measurements at every fourth sweep and reunitarization after every second sweep.

Runs reported here range in length from 1×10^4 to 3×10^4 sweeps. Except for the first run at each lattice size, the lattice was intialized to the result of a run at a nearby value of β on the same size lattice. The first 2000 sweeps at each coupling were discarded. This number, 2000, represents a compromise between the ideal of a truly independent start and the scarcity of data.

New results are shown from three Monte Carlo runs in Fig. 1. These results are from $19^3\times14$ lattices, the largest size we have studied. In the first run we are below the transition point in the confined phase. The second run shows the system near the transition point with the coexisting confined and deconfined phases. In the third run we are above the transition point in the deconfined phase.

In an attempt to clarify scatter plots such as these, we have made "blocked" scatter plots in which we averaged the Polyakov loop over several successive measurements. The idea is that we are averaging out the high-frequency (in Monte Carlo sweeps) scatter in the data but leaving the much slower movement of the system among the different available phases. Figure 2 shows the same data as in Fig. 1, where five successive points have been averaged.

The coexistence of the confined and deconfined phases over runs of extended length (20,000 to 30,000 sweeps were typically required to study the system close to the transition point) and the jump in the order parameter P provide evidence that the transition is of first order.

The width $\Delta\beta$ of a temperature-driven, first-order phase transition is expected to scale as

$$\frac{\Delta T}{T} \approx \frac{1}{sV} \tag{6}$$

where $V = n_s^3$ is the spatial volume and s in the latent entropy of the system.[11,12] For small values of the jump in the "magnetization" P the latent entropy will be proportional to the jump in P at the transition point. Near a first-order phase transition we also anticipate a shift in T_c as a function of the volume V with the same scaling law given by (5).

Estimates of T_c can be made from visual inspection of scatter plots such as those in Figs. 1 and 2 and plots of the magnitude of the order parameter versus sweep number. Naturally it would be useful to have a quantitative measure of the degree of confinement or deconfinement of a finite-size lattice. In an attempt to do this we have studied the dimensionless quantity

$$\pi = \langle \cos[3\, arg(P)] \rangle \tag{7}$$

which we call the "triality." Here $arg(P)$ is the phase of the order parameter, and the brackets indicate an average over the Monte Carlo samples. The triality depends on the coupling g, the temporal size n_t, and the spatial size n_s. It vanishes in the confined phase and is one in the deconfined phase on an infinite spatial lattice. The triality can be used to extract the critical temperature from measurements on lattices with finite spatial volumes. One can measure the value of g for which the triality passes through an arbitrary value on lattices of different spatial sizes and extrapolate to infinite spatial size using the scaling of (5). This procedure works well for $n_t = 4$ and $n_t = 6$, where we have accurate data for a range of spatial volumes, and we have verified that the results are independent of the value of the triality used to define the phase transition. On the larger lattices on which we are reporting we have data on such a small range of

Table I. The Measured Critical Values of $6/g^4$ for Different Values of τ^{-1} and Spatial Size

τ^{-1}	N_s	$6/g^2$
8	13	$6.02 \pm .02$
	19	$6.02 \pm .02$
10	17	$6.15 \pm .03$
12	17	$6.32 \pm .03$
	19	$6.32 \pm .03$
14	19	$6.47 \pm .03$

volumes that this approach has no real advantage over visual inspection of the scatter plots for determining the critical temperature. We have still found it useful in estimating the errors in our results to calculate the statistical uncertainty in the value of β at which the triality reaches some arbitrary value. Table I summarizes our results of a large number of Monte Carlo runs.

Estimation of the statistical errors is difficult because of the limited amount of data and because the T_c estimates come largely from visual inspection of scatter plots and time histories. Even our longest runs contain only a handful of tunnelings among the different phases, so one cannot sensibly divide the runs into many independent subsets as we would do if we had much more data. Thus our quoted errors represent the range of β over which the scatter plots appear to change from clearly confined to clearly deconfined.

A more detailed discussion of our simulation and the analysis is in preparation.[13]

CONCLUSION

The main physics result of our work is depicted in Fig. 3. The measured values of T_c/Λ are plotted there in the range from 8 to 14 for τ^{-1}. For the sake of clarity points at $\tau^{-1} = 2, 4, 6$ from Ref. 8 are also included. Where the relation between g_c and τ^{-1} is that predicted by two-loop perturbation theory, this graph will be a horizontal line. The height of the line gives the constant of proportionality between Λ and T_c.

The following remarkable structure appears: after apparent early scaling between $5.1 < 6/g^2 < 5.7$ there is strong scaling violation in the range $5.7 < 6/g^2 < 6.10$ and, finally, asymptotic scaling is observed for $6.15 < 6/g^2 < 6.50$. This appears to be an earlier onset of scaling than

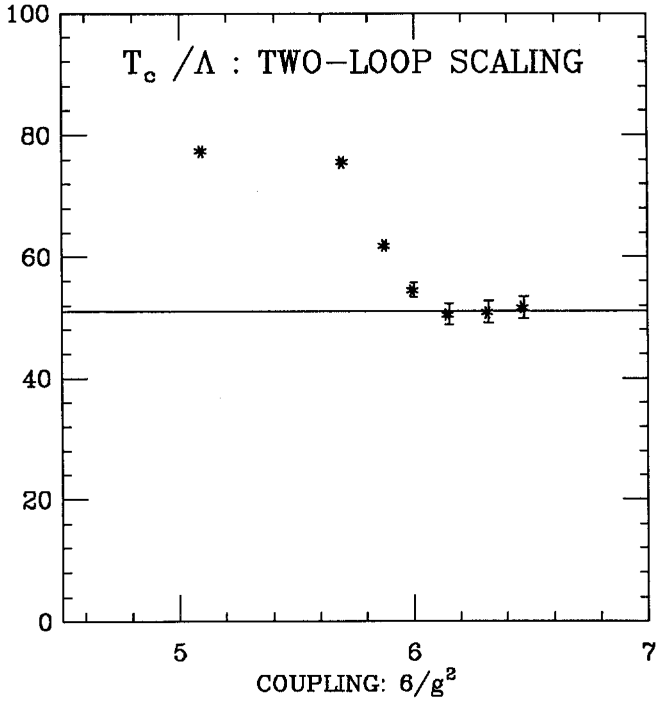

Fig. 3. The onset of scaling in the deconfinement temperature.

reported from recent Monte Carlo renormalization group[14,15] and ratio test methods.[16]

The onset of scaling is at much weaker coupling than early optimistic expectations. This means that either a large increase in computer power or a substantial improvement on Wilson's lattice action is needed for practical calculations of hadron properties. However, this work does provide evidence that Monte Carlo calculations with $\beta > 6.15$ on sufficiently large lattices can provide believable answers for continuum quantities in pure gauge QCD.

After completion of our work we learned about some related recent results with somewhat similar findings.[17] Though the qualitative conclusion of Ref. 17 on the onset of scaling is similar to ours, the differences in details will require further clarification.

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